

Boundaries and Groupings: A Survey of Mathematical Concepts in 18th/19th Century German Aesthetics

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Introduction

The rigorous study of art, from both the critical and philosophical perspectives, faces two complex tasks. The first is the commentary on things of one type of material representation by things of another type. Most often this means the discussion in words of art expressed in something other than words (or in the case of poetry and literature, one use of words discussing another use of words). The second, following from this first, is the balancing of qualitative and quantitative approaches to describing art or outlining its boundaries. Curiously, this second is most often done in words as well, except by a new wave of 20th and 21st century scholars who are beginning to employ mathematics in the service of understanding art.

It is not the goal of this discourse to explain in detail the particular approach of any of these contemporary scholars or to suggest that it is always superior to more verbally oriented methods. Instead, what follows is the application of a mathematical perspective to comparing concepts expressed in the aesthetics of Gotthold Ephraim von Lessing, Immanuel Kant, and Friedrich Nietzsche. The strength of mathematics lies in its power to explain structure independent of content, but in attempts to use language to describe art which perhaps describes something else, we suppose here that practical

demands on the scholar within their temporally and geographically constrained tradition likely preclude the sufficient study of the most advanced mathematics of the time: the mathematics that might be required to actualize this power. Thus, we contend that it is useful to look back at aesthetic theory of the past with the mathematical tools of the present, released at some point from the highest echelons of academic mathematics, reserved for pure mathematicians until they surpass it and leave it behind for application.

Note that both art and mathematics themselves contain the processes of selection, comparison, categorization, synthesis, and their opposites. So the following will show examples of analysis by mathematical techniques that mirrors the process of artistic creation. Is a system of art analysis a work of art in itself? Perhaps, especially if we agree with Nietzsche's broad claim that "one must give value to their existence by behaving if one's very existence were a work of art" [4]. From this perspective of Nietzsche, consider the following:

1. Life = work of art, Living = artistic practice (to be done well)
2. The experience of art is a subset of the possible experiences of life.
3. Aesthetic philosophy and art criticism are particularly deep experiences of art, a subset of the whole of possible experiences of art.
4. Therefore, aesthetic philosophy and art criticism are artistic practices in themselves.

We will henceforth operate within this framework established by Nietzsche, using the aforementioned common principles of art and mathematics to create a new framework for analyzing art in a way that collectively applies the concerns of Lessing and Kant as well. Curiously in doing so, we will be acting in the capacity of Lessing's "critic" one further level of abstraction away from how that critic supposedly acted [3]. This practice is subsumed by Nietzsche's conception of the artist, so we see the union of artist and critic in selecting and applying a general procedure.

The work of Immanuel Kant comes in, with yet another twist in this conversation between mathematics and art. While Lessing and Nietzsche are concerned with categorization and contextualization procedures respectively, which are related, Kant directly tackles the roles of the quantitative and qualitative in the navigation of these categories and contexts (we call them "boundaries" collectively). First, we will discuss Lessing's examination of art forms according to their relationship with the physical constraints of time and space and the mathematical properties those admit. These properties align well with Nietzsche's analogy between life and art, as constraints on the former necessarily imply constraints on the latter [4]. Kant then becomes relevant at the phase of navigation, in which the human experiencer reacts to the boundaries and groupings over space and time. Kant's beautiful and sublime are particular conditions of art that arise from perceptual correlates to the experience of the interior and exterior of mathematical boundaries, the finite and the infinite. So in summary, we build the system by Lessing's principles and their emergent properties, navigate it by Kant's categories of experience, and extend it to its broader parallels by way of Nietzsche. What will emerge

is a robust understanding of artistic practice explained at the various levels explored by these scholars, reduced to a common language that can both explain and create.

Lessing's Three Experiencers

Let us start from the mathematical logic. The number 4 can less controversially be factored into 2×2 than the ontology of musical dynamic categories can be split into the dichotomy of loud/soft. Thus depending on the demands of the discipline for particular forms of justification, the relationship between the quantitative and the qualitative varies, and the best synthesis of these for a successful explanation depends upon that which is being examined. Aesthetic justification comes from the senses and the emotions where mathematical justification supervenes on logical combination of properties and the sets they delineate. Since art is a material manifestation, and we do not first experience the ideas motivating it, but rather this material outcome, we move from mathematics to physics, the examination of the mathematically organized phenomena experienced in the natural world.

Lessing divides the experiencers of art into three types: the amateur, the philosopher, and the critic. The amateur has a refined sensory experience, the philosopher inquires about the nature of artistic experience and the sources of its power, and the critic compares trends in different art forms and makes prescriptions from his or her findings [3]. It is important to consider these types of art experiencers because they immediately frame the audience experiences as differing in the extent to

which they are qualitative or quantitative, which we will later see Kant examining in detail as properties alone.

Of the three types mentioned, Lessing says “the two former could not easily make a false use either of their feelings or of their conclusions. But in the remarks of the critic, on the other hand, almost everything depends on the justice of their application to the individual case” [3]. Consider now the differences in exactitude of the demands placed on each of Lessing’s three types of experiencers. The amateur, who refines his capacity to discern over repeated exposures to art, Lessing calls “a man of delicate perception”. How might we have learned in Lessing’s time that a man’s perception is delicate? We did not have brain imaging techniques that operated on mathematical systems and outputted quantitative measures of activity that suggest delicacy. Instead, we would have had to rely on his words, the depth and specificity of his verbal expression about his experience of the art. So here we are again, faced with art itself as we try to analyze art. In fact, we are faced with our experience of the artistic effort undertaken by the man who expresses his experience of art by way of artistic practice of speech or prose.

The relationships in the structure and meaning of the words have to somehow convey a message about the message the art conveyed in structure and meaning of perhaps some other phenomena like textured and colorful paint spread out over the space bounded by a canvas. Now we come upon the perspective of the critic again, but applied to description of artistic experience, subsuming the description intended by the art itself. The critic, as Lessing said, can be more objectively judged. Why is this? The

critic is engaged in comparing and contrasting particular dimensions of artistic practice in different media. At its most basic level this means taking two concepts and labeling them collectively as either alike or different. This is a much clearer categorization than the subjective one made by the amateur in verbal description of how the art makes him feel. “Alike” and “different” are relative terms, whereas descriptors like “beautiful” or “terrible” or “ugly” as Lessing later addresses, introduce new categories of which instances of experience either are or are not a part. If they are not, they are left undescribed, as the system of adjectives used to describe art is too variegated to be a closed one like classification as “alike” or “different.”

We can describe this quite succinctly in mathematical terms. A likeness and difference form a closed group with discrete entities. Beauty, ugliness, and terribleness, do not unless we can find a reason not to think of graceful, obese, and horrible as synonymous with or subsumed by the first three [3]. We chose the last three example descriptors intentionally for their respective relationships with the first three. Categorizations also run into the problems of kinds vs. degrees and of parts vs. wholes. Is grace a distinct descriptor that need not involve beauty, or is it a necessary but not sufficient part of beauty? Essentially we are looking for the answer in the following framework...

$$\text{grace} \not\cap \text{beauty} \parallel \text{grace} \subset \text{beauty} \parallel \text{grace} = \text{beauty} ?$$

The last relation mentioned is included as the extreme case of synonymy. If we decide on either of the first two, we are making a conceptual distinction. If we decide on the last, we are supposing a lack of conceptual distinction, which then raises the

question, why have two different words to explain the same thing? Is it due to a structural mismatch between language and the particular art form being discussed? Is it a deficiency in the amateur's delicacy of perception to not know the difference between beauty and grace and to use them synonymously? If we accepted the middle concept, $\text{grace} \subset \text{beauty}$, where is the boundary of grace at which other features achieve sufficient salience to require the experiencer to express the more general experience of beauty. Keep this distinction in mind, as later we will see how Kant may help with it by way of a different mathematical framework in his description of experiencing the sublime [2]. For now, we turn from the experiencer to the artist and the art itself.

Lessing's Physics and Combinatorics of Art

In his 1766 work "Laocoon: An Essay On the Limits of Poetry and Painting", G.E. Lessing advances a medium constraint thesis in differentiating between possible modes of expression in the two art forms named. To do this he enlists the concepts of space and time, ideas firmly grounded in mathematical systems that explain art in the way Lessing does and life in the way physicists do. "Signs must have a definite relation to the thing signified", Lessing says to characterize the goal of his framework [3]. Painting uses "figures and colors in space" whereas poetry uses "articulate sounds in time". The former has bodies as its subjects and the latter, actions instead. In space, objects can exist side by side, but they remain unchanging in themselves over time. So the direct experience is art over space. But what of time? Is it not part of the experience? Note that I said the objects remain unchanging "in themselves". The perceiver of course, may

contemplate an object in a different context, in a larger set of roles related to the one it is actually fulfilling in the painting, etc. In fact, with this uncharted temporal context, the viewer can engage in what Lessing calls “free play of the imagination”, which he considers a crucial part of the artistic experience. With bodies on display frozen in time, the free play occurs on actions over time [3].

In the case of poetry, the situation is reversed. Poetry occurs over time, as words are read in sequence and can describe actions. Free play of the imagination then, is left to contemplation of the bodies, as if they are ever described, this is followed by a shift in the subject matter over time that leaves further existence and evolution of the previously noted body unexplained by the work itself (Lessing 81). Now, what of other art forms beyond painting and poetry?

Perhaps Lessing chose painting and poetry for their apparent straightforwardness and relative simplicity when viewed according to their relationship to space and time. The content of poetry does not take up actual space, and the content of painting does not evolve over time. Furthermore content of poetry is experienced narratively and the content of painting is experienced visually. Other more complex art forms, like theatre for example, involve the evolution of both narrative and visual content over space and time: movement of visible bodies as they perform understandable actions.

From this the question arises: of the set of phenomena expressed by the work of art, how much/many can the experiencer attend to and how often/long can the experiencer attend to it? Faced with the diverse sensory experiences over space and

time in a staged work, we return to the artistic and humanistic practice of selection. So here we propose an evolved version of Lessing's model, described succinctly by the following equation:

$$\text{Experience} = \text{attention}(\text{space}) + \text{attention}(\text{time}) - \text{attention}(\text{space, time}) - \text{attention}(\text{time,space}) + \text{attention}(\text{whole work})$$

We view instances of spatial representation in the art as a set of things to be attended to, and instances of temporal representation the same. So these must be added together to get closer to the whole experience. We then must subtract the instances in which both were present in either order, because the one more strongly attended to has already been accounted for. Finally, we add back in the experience of the whole work in order to account for the emergent properties that may inhere in some of these contemporaneous experiences of spatial and temporal representation: the unordered experience of these two rather than the ordered which we subtracted out.

Now of course, there are different aspects of the presented work itself to consider spatially or temporally, and these too are chosen over both space and time. If this is the case, the above equation has settled the matter of kinds but not the matter of degrees. With this in mind, we must include all combinations of subsets of space and time presented that can be contemplated, and the repeated aesthetic calculations that inform the last factor, attention(whole work). So we have all the possible subsets of {space, time} as well as their infinite possible orderings given the lack of constraint on the number of terms. Faced with this evolution of experience and lack of known constraint on the number of terms (this might be answered normatively by neuroscience in looking

at the information the working memory can hold, and the abstractions made over time in order to craft a whole interpretation). The previous parenthetical of course privileges the experiencer over the artist, which we are not necessarily in favor of arguing. The artist does the same thing according to his or her own abilities, and the communication of a message depends upon the harmony of the processes undertaken by the artist and the audience. Regarding these processes, the literature says more about the audience experience, but combinatorics on Lessing only deepens the problem, it does not solve it. For that we must turn to Kant's beautiful and sublime.

The Aesthetic Calculus of Kant

As it is often used in philosophical work, "calculus" refers simply to "method of calculation". Here we refer more specifically to the branch of mathematics that deals with the evolution of structures in space over periods of time. This follows nicely from Lessing's use of those to categorize and compare expression, as we now look at how Kant indirectly uses them to frame two types of experience. Note now though, that we are looking at continuous phenomena rather than discrete as in Lessing's framework.

According to Kant, the "beautiful" refers to a judgment about the form of the object, or its limitations. The "sublime", on the other hand, refers to formlessness or limitlessness contained within a notion of "totality" [2]. In what mode of physical understanding do these limits occur? What about the limitlessness? This surely recalls Lessing, who would say that the limits occur according to the boundaries inherent in the

medium, and the limitlessness is the free play of the imagination to relate what was not explicitly presented. Here, Kant has appropriated the mathematical concept of a set and its complement in order to define terms used to explain description of aesthetic experience in the same way Lessing employed that concept to talk about boundaries inherent in production and expression.

Oddly, Kant says that “the satisfaction [from beauty] is connected...” with the “representation of *quality*” and the satisfaction from sublimity is connected “with that of quantity” [2]. We are prepared to call this odd in light of the combinatorial model of Lessing’s medium constraints, because we can find merit in synthesizing these concepts in analyzing art like Kant does, and then selecting, comparing, and categorizing the analytical results. The question then quickly arises, are these the only two concepts necessary? Should we rename the products of their various syntheses? We will not argue one way or the other here, our purpose is to show the complexity and provide a model that preserves the terms without denying the extent of their interrelations.

The beautiful is associated with quality, because by way of its association with form, we can assume we are looking at a work of art from the outside, its appearances. So our perceptual capabilities operate on the outside of the object and see its form, its limits in space and time. What is this object like? To answer that, we’re likely to talk about form.

The sublime on the other hand is associated with quantity because of its relationship to the supersensory: Kant sees the sublime as the extremes of magnitude

that exceed our capacity for sensory apprehension [2]. But why must the quantitative be restricted this way? Magnitude is synonymous with degree, which is distinguished from kind when looking at a collection of objects. Might many kinds also overwhelm the senses?

Now returning to the beautiful, form too admits analysis by both kinds and degrees. If the beautiful consists in forms, in limits, and Kant says it is achieved qualitatively, then how can we differentiate between two limits of the same kind? For example, let's say woman A is beautiful in part because of her agreeable height, which falls within the exemplary beautiful range of 5'4" to 5'8". This one is 5'6", but her friend is 5'7". The relevant difference being examined is one of degree. Does that inch really matter? Perhaps it does, in which case we are done, we have shown that beauty also depends on degree, on the quantitative, on the precise height. If not, perhaps we're saying any woman over 5'6" will do. In this case we have still used a quantity even when assuming Kant's framework. The chosen quantity is in fact what determines the form of this way of perceiving feminine beauty, which we can see from this again abstracts toward viewing aesthetic perception as an art form in itself. We have now constructed an evaluation procedure in which according to our own preoccupations, over the quantity 5'6" and under the quantity 5'6" are now kinds, though the height measurement itself is a degree. We use the limits to frame the unlimited.

Again with the language, we have an explanatory predicament. For this we can turn to the mathematics of change. Kant would view a curved boundary as a limit, but not all curved boundaries are alike. Some curve more, some curve less. Over time

viewing segments of the curve from the initial point to the terminal point, the space underneath it changes. We cannot simply call it one curve if there is not some uniform way of understanding it. If there is, then from that vantage point we would not view it as change. The space contained under the curve then, consuming the viewer with the experience of beauty, is represented by the familiar:

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a) .$$

In the above equation, a and b represent the starting and ending times, F represents the height of the function at a particular time in [a,b] (value of the “beautiful” formal boundary), and dx represents the width. F(a) is subtracted from F(b) to account for the limit of that form, i.e. it exists between a and b. All $x > b$ and $x < a$ are part of the limitlessness outside the form from which that form is being examined. This shows that the quantitative is a clear component of the diverse possibilities of form. Perhaps if $f(x) = Ax$, a line, we would think of that as one kind of form, whereas if $f(x) = Ax^2$, a parabola, that is a different kind of form. The issue is much more complex than one would realize without the mathematics, simply assigning “sublime” to quantitative and “beautiful” to qualitative. Note that we are not casting doubt on the value of what Kant did assert; rather, we are showing the limits imposed by the system on the unlimited (or less limited?) complexities of aesthetic experience.

Now, returning to the sublime... suppose we have a multicolored quilt. “Multicolored” is in part a quantitative description, referring to a number of colors on the quilt that will impact one’s perceptual experience. But how do we divide up these

colors? What if they are all shades of blue? That is all one color but is likely more than one experience. Would a quilt of many shades of dark blue squares of the same size be a sensory experience of the same magnitude as a quilt of dark blue and bright yellow squares of different sizes? I say no, but I must first address the counterargument that this too is a quantitative matter related in the following ways:

1. Shades of blue, equal sized squares: Quantity(Quilt A) = # of different shades of blue.
2. Highly contrasting blue and yellow, equal sized squares: Quantity (Quilt B) = # of patterns of two colors
3. Highly contrasting blue and yellow, different sized squares: Quantity (Quilt C) = (# of patterns of two colors X (# of patterns of differently sized squares)).

The size of the squares is precisely a matter of quantity. The shades of blue could be mapped onto a quantitative model, but they could also be seen as differences in quality that we can describe, i.e. light, dark, bright, mellow, etc. Can we really say that all viewers of the quilt will regard shades of blue as different degrees, different quantities of blueness? Can we say instead that they all regard these shades as different kinds, different qualities... periwinkle, turquoise, navy, etc...?

While it may be clear what Kant assumed in his model, it is not clear that this does justice to properly balancing the complexity of actual art or of human contemplation of art. In fact, recalling Lessing, we have different sorts of people contemplating the art: the amateur, the philosopher, and the critic. With these subtle

distinctions and their inherent assumptions about how aesthetic components will be categorized and explained, we cannot assume that viewers of widely different experience and preoccupation will make such clean and consistent distinctions between these things. The artist, though perhaps tending toward the more educated and experienced, also navigates these distinctions of space, time, degree, and kind in producing something to be experienced as in part beautiful and in part sublime. If the sublime really is a matter of quantity, then the integral definition used for beauty (equation above) provides a richer explanation of sublimity in the same way, allowing for its variation over space and time. If it is not, we are back to a matter of combinatorics on quality:

$$(\text{Kind A}) \times (\text{Kind B}) \times (\text{Kind C}) \times \dots = \text{Aesthetic Experience of Work}(\text{A, B, C} \dots)$$

Note that in a similar fashion as we've seen before, this complexity is deepened by the consideration that some proper subset of $\{A, B, C, \dots\}$ could also be perceived as a unit, so we would then have to think about variations of perceptual focus on A, B, C, AB, AC, BC, ABC... again deepening the manifold hierarchy of instances of the beautiful and the sublime. The more realistic equation, as we let the artistic representation of the thing tend toward the thing in itself (or in Nietzsche's terms to be soon addressed, we let art blend into life), is as follows:

$$\text{Kind A} \times \text{Kind B} \times \text{Kind C} \times \dots - \text{Kind(A, B)} - \text{Kind(A, C)} - \text{Kind(B,C)}\dots + \text{Kind(A, B, C)} + \dots = \text{Aesthetic Experience of Work(X)}$$

Nietzsche on Art and Life or.... $\text{Art} \cap \text{Life} = \text{Perspective}(\text{Space, Time})$

At this point we have accomplished two things. First, we have provided mathematical models for understanding the aesthetic phenomena of operation of ideas within space and time, and according to order and magnitude. For these we used a blend of principles from combinatorics and calculus, as we examined the artistic procedures of grouping elements, defining boundaries, and accounting for change over time. While Lessing's model and Kant's model both do these things in part, there were layers of complexity that were not addressed that call some of the strict definitions and separations of ideas into question. As one can imagine by looking at some of the above equations and considering their ranges for large numbers of variables or vast differences in quantities, the prescription of how to view the work of art to be analyzed gets complicated quickly. How many beautiful forms are there really? Given the wide range of possibilities of magnitude and the infinitude of possible units of measure, how many paths to the sublime overwhelming of the senses are there really? Many? Infinitely many? What constrains it if anything? Contemplate those questions with the following relation in mind:

$$\text{Art} \subset \text{Life} \ \&\& \ \text{Experiences}(\text{Art}) \subset \text{Experience}(\text{Life})$$

The discussion so far recalls Friedrich Nietzsche in his 1873 “On Truth and Lie in an Extra-Moral Sense”, in which he says, “the various languages placed side by side show that with words it is never a question of truth, never a question of adequate expression; otherwise, there would not be so many languages. The “thing in itself” (which is precisely what the pure truth, apart from any of its consequences, would be) is likewise something quite incomprehensible to the creator of language and something not in the least worth striving for” [1].

So in considering Nietzsche in relation to the mathematics of Lessing and Kant, we come up against the broadest boundary yet, between art and life. As we have artfully proceeded so far, extracting the structure of concepts about art from one philosopher and applying them to those of another, and doing the same with the experiencers and the artists, we can see where Nietzsche has done or suggested the same thing. In the quote above for example, his “creator of language” is taking expressions from the subset of possible utterances to form a language. This language, as Nietzsche indicates, deviates from truth in the way it affixes constant labels to concepts and observations. Nietzsche’s argument for this, also in the quote above, is that if one language were to express truth, we would not need other languages. This maps directly onto the reason for the infinitude of prime numbers in mathematics, shown by the equations below:

First we will construct a number:

$P = P_1 \times P_2 \times P_3 \times \dots \times P_n + 1$, where P_1 through P_n constitute all of the prime numbers (pretending there are finitely many)

Since P_1, P_2, \dots are all of the primes, P cannot be prime, therefore it must be divisible by at least one of the primes listed.

But P_n was our largest factor, and when you divide the right side of the equation above by it, the remainder is in fact 1, thus our claim that P_1, P_2, \dots, P_n constitutes all of the primes, and that there are finitely many, is false.

Why does this matter for Nietzsche's relation of art and life? Let us now look at the whole number system as we would a language, admitting all the possible interrelations of quantity and quality, and how they can be used to comment on kind and/or degree. Since we were already examining the primes, this simply entails including the notion of the composite numbers along with the primes as $P + C = N$ (which denotes the natural numbers). Similarly we can say $(\text{Art}) + (\text{Non-Art}) = \text{Life}$. Nietzsche says that we ought to treat existence as an art [1]. We tend not to do this though, as evidenced by the act of labeling something as art, which entails the existence of non-art unless we are making a trivially broad assertion. Any meaningful judgment suggests a division of some body of ideas into those judged to meet a particular criteria and those judged not to meet it. In choosing a way to live, we implicitly label all other ways we consider as not the way we ought to live. Here we have reduced the problem to three parts: 1. The infinitude of the primes / ways to live 2. The incompleteness of the primes, i.e. they are only a subset of the natural numbers / the

incompleteness of considerations of how to live before committing an instance of living, and 3. The boundaries constructed by selection of primes / Nietzsche's perspectivism [1].

Concluding Remarks

For the purpose of showing a continuity of structural problems, we have reduced work on three different areas of aesthetics to parallels in mathematics. Separating out the portions of aesthetic experience and analysis that follow mathematical patterns allows a deepening of understanding without arguing for or against any of the concepts admitted into the framework, and it leaves behind the part of the experience that is not mathematical. An act of analysis then, is simply a shifting of perspective as one set of content is matched with one practice of explaining form, and we can then move onto the next, using the previously problematic material as a new tool for the same procedures of selection, comparison, categorization, synthesis, and their opposites at the next structural level.

The procedures themselves are applicable to both art and life. As was shown by example, they are also recursively applicable to the analysis of art and thus of life. Finally beyond that, if we analyze methods of analysis themselves, we are admitting the sorts of verbal procedures we have just mathematized. So where there is an analysis of art being proposed, there is also an art of analysis being proposed. Where artistic practices are meant to reflect life, living practices can form an art. So with the broadest layer of complexity reached, let us summarize the others again.

We begin with the constraints from the physical world as they impose on the media from which works of art come. In fleshing those out with mathematical techniques so as not to miss any possibilities, we reach the body of all expressive potential constrained by that which originally informed it (the physics). Then we have the set of all works to look at, and the effects they may have on their audience. The mathematics of these align with the mathematics of the media, and reflect the way we engage with art. But how do we engage in general? As artists, with life as our medium, according to Nietzsche. From here we realize that in the choice of constraints, of organizing principles, of methods for refining these, we are faced again with the constraints of the medium: this time, the human condition. From here it seems futile to continue in the same manner in hopes of reaching a truth about art or life, given the structural overlaps and inseparability from choice. Perhaps then, we ought to either move onto a different approach or declare the understanding that it is structure over rather than expressed content, where real parallels appear between the artist, the art, and the life both are contained within.

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